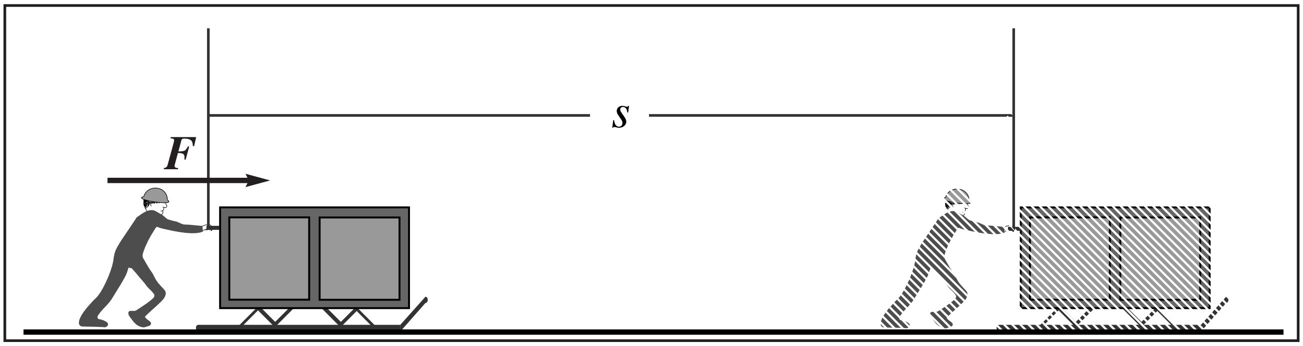
Work and Energy

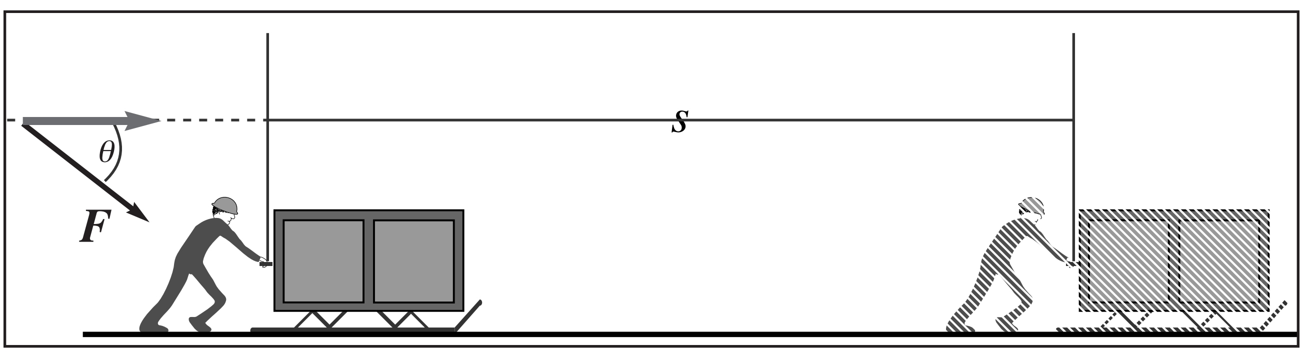
In a mechanical system, the **work**, W, done by a force upon an object is equal to the product of the component of the force in the direction of the object's displacement and the magnitude of the displacement. The rule is "work equals force times distance", but remember that only the component of the force in the direction of the displacement does any work.



For example, the man pushing the cart above exerts a force in the same direction as the displacement of the cart, so in the illustration above the work does equal "force times distance."

Below, however, the handle of the cart is not so conveniantly placed. The man must exert a force that is partially downward to push the cart forward. For this case, a more general expression is useful:

$$W = (F \cos \theta)s$$



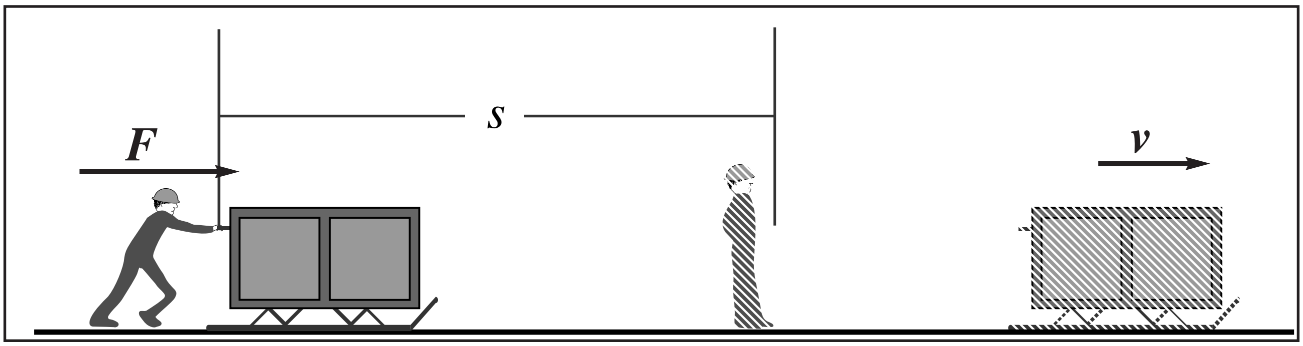
The unit employed in quantifyng work is the **joule**. One joule is equivalent to the work performed by the application of a force of one newton through a distance of one meter.

$$1\ \mathrm{J} = 1\ \mathrm{N} \cdot \mathrm{m} = 1\ \mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Kinetic Energy

Why is work important? Because work represents the exchange of **energy** in a mechanical system. There are many forms of energy, but what relates them all is that when they are transformed from one form to another, the total amount remains the same. One type of mechanical energy important to our discussion is **kinetic energy**, K, which is the energy possessed by an object because of its motion. In the illustration below, by pushing the cart through the distance s, the man converts the chemical energy in his muscles into the kinetic energy of the cart. The amount of energy transferred to the cart (assuming there is no friction) is equal to the "force times the distance," or the work performed.

$$W = K_f - K_i = \Delta K$$

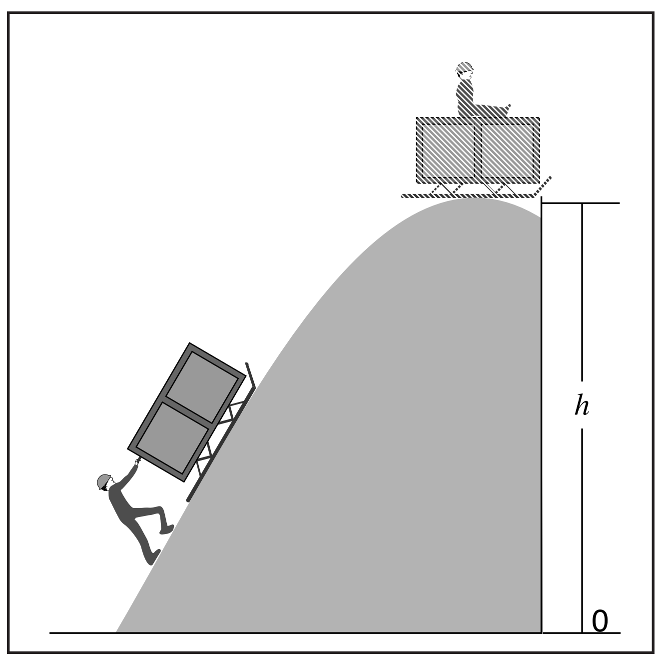


If the mass and speed are known, the kinetic energy of an object may be quantified:

$$K = \frac{1}{2}mv^2$$

Potential Energy

In a mechanical system, work may also impart energy in another form. An object possesses **potential energy**, U, because of its position with respect to a standard. When we lift an object against the force of gravity, we are storing energy in the gravitational field, energy that may be transformed into kinetic energy when the object falls back through space. The work necessary to lift an object to a certain height, h, is simply equal to the weight of the object multiplied by that height ("force times distance). The potential energy thus stored is equal to this amount of work:

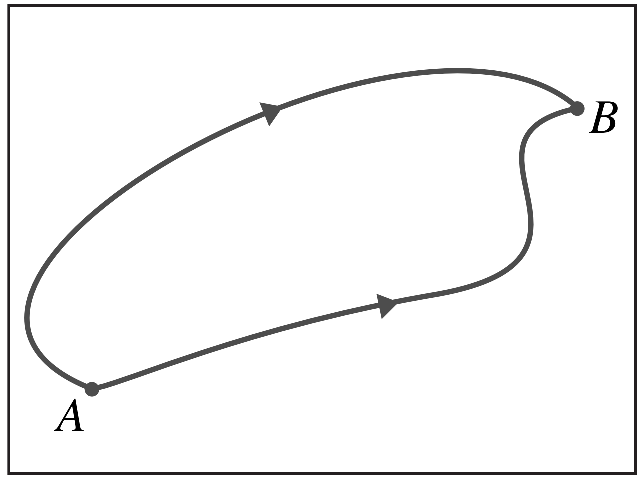


$$U = mgh$$

In the above expression, the potential energy is taken to be zero on the ground. The choice of reference point is not really important, because it is only the *change* in potential energy that is significant.

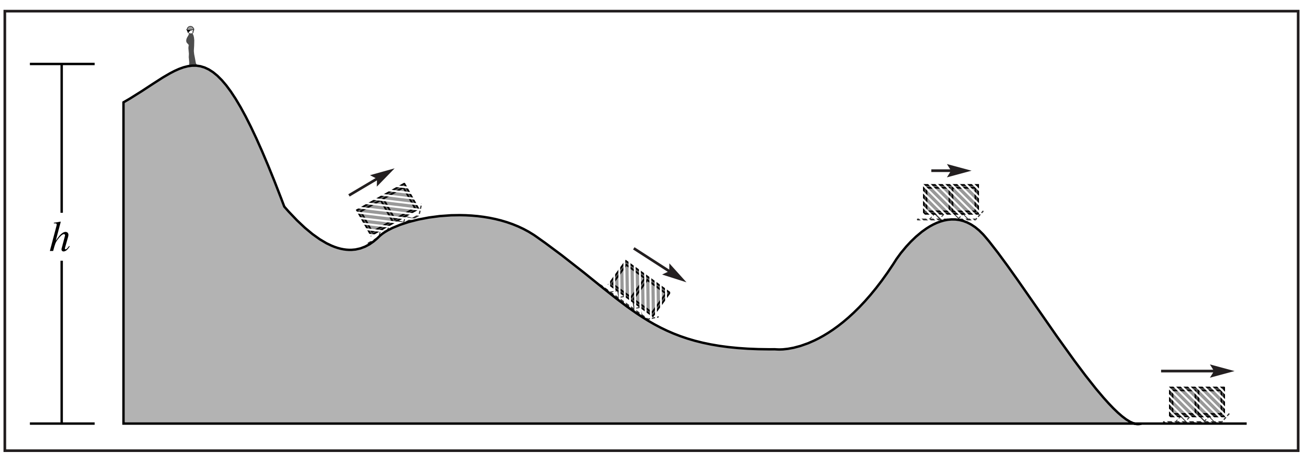
Conservative and Nonconservative Forces

If the amount of work done by a force acting on an object moving between to points is independent of the path taken between the two points, the force is **conservative**. Gravity is a conservative force, while friction is nonconservative. If the friction force were the same for the two paths at right, more work would be required to take the upper path, which is longer. If the work were done against gravity in a frictionless environment, then the work required would depend only upon the initial and final positions A and B. It follows in this case that if the object is moved around a closed path and returns to its initial position, the total work performed is zero.



The Conservation of Mechanical Energy

If the only force that does work in a system is a conservative force, the total energy remains constant. If the kinetic energy increases by a certain amount, the potential energy must decrease by that amount and vice versa. Prior to releasing his (frictionless) cart to roll down the hills in the illustration below, the man has put energy into the system by doing work to push the cart up to the height h. The energy at that point is completely in the form of potential energy, mgh. From that point on the total energy never exceeds or falls below that amount. Rolling up the hills that follow, the kinetic energy is decreasing and potential energy increasing. Rolling down, the kinetic energy increases and potential energy decreases. At the end of the hills, when the cart is at height 0, the amount of kinetic energy possessed by the cart is equal the initial potential energy, which equals the amount of work the man had put into the system.



The mathematical expression for the conservation of mechanical energy is as follows.

$$K_i + U_i = K_f + U_f$$

Power

Power is the rate at which energy is expended or work done. If a force performs work on an object for a certain time interval, the average power during this interval can be expressed:

$$\bar{P} = \frac{\Delta W}{\Delta t}$$

Power, therefore, is measured in joules per second, which is called a *watt*.