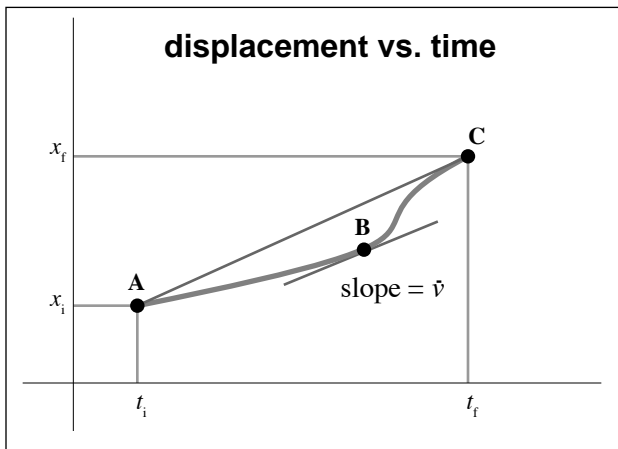


# Kinematics

Kinematics is the portion of mechanics concerned with describing motion. At this point we are not addressing the causes of motion, i.e. forces, but are only supplying ourselves with the language to describe motion in terms of space and time.

## Motion in One Dimension

A particle in motion changes position. The difference in position between two points in its path is termed the displacement of the particle for the time period involved. To understand the relationship between the concepts displacement and velocity, consider the following graph describing the motion of a particle in one dimension in terms of position and time:

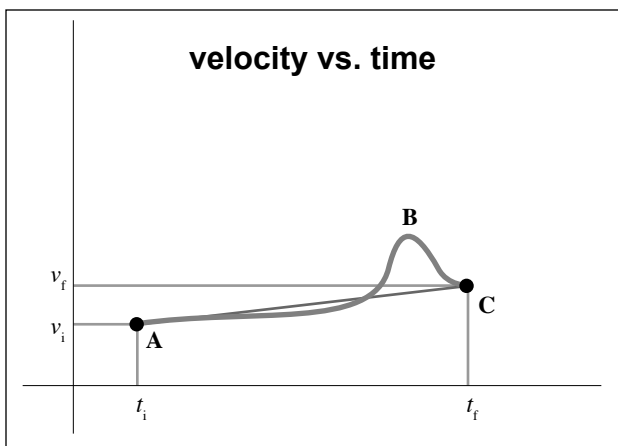


The slope of the line connecting the two endpoints A and C represents the change in displacement between those two points divided by the time taken for the particle to travel between them. This is the average velocity of the particle over that time period.

$$\bar{v} = \frac{x - x_0}{\Delta t}$$

The slope of the line tangent to point B represents the instantaneous velocity of the particle at that point. Instantaneous velocity refers to the rate of change of displacement at an instant of time or at some point. In general usage, the term velocity refers to instantaneous velocity.

We may also represent the motion of a particle by graphing its velocity with respect to time: Here the slope of the line connecting the two endpoints A and C represents the average acceleration of the particle over that time period. This is the change in velocity divided by the time taken.



The instantaneous acceleration of the particle refers to the rate of change of velocity at an instant of time or at some point. In general usage, the term acceleration refers to instantaneous acceleration.

$$\bar{a} = \frac{v - v_0}{\Delta t}$$

## The Equations of Kinematics

The formulas of physics are a kind of language, enabling the interpretation of physical events in a quantitative manner. Over the years as physical sciences education has lessened its emphasis, in general, on the memorization of formulas, this has increased the necessity for students to be able to approach a particular physical problem and form conceptual interpretations. However, it would be a mistake not to study the formulas. Formulas are compact expressions of the ideas of physics. When encountering a formula, imagine the real situations that it represents. Imagine the value of one term increasing or decreasing and conceptualize the effect on other elements in the expression. Ideas like velocity, force, temperature, or wavelength are constructs which we utilize to interpret our universe. Physical formulas place ideas such as these in relation to each other. Understanding the formulas goes toward understanding the concepts. This type of understanding is different than memorization.

There are, however, four equations in kinematics which really should be memorized. Note that these expressions allow one to interpret motion characterized by constant acceleration.

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

*This equation simply states that the displacement of an object undergoing constant acceleration equals the average velocity multiplied by the time of motion. Compare and contrast  $\frac{1}{2}(v + v_0)$  as an expression for average velocity with the one from the previous discussion,  $(x_f - x_i)/\Delta t$ . Do you see why the expression from the previous discussion applies to the more general case where acceleration may not necessarily be constant?*

$$v = v_0 + at$$

*To restate this expression verbally: the velocity at time  $t$  equals the initial velocity plus the acceleration multiplied by the time of motion. The mathematical expression is much more compact, but it is a good exercise to verbalize a formula to yourself in order to bring it to your understanding in a familiar way. Also, remember that the formulas represent something real! For example, the units of acceleration are  $\text{m/s}^2$ . One can say, "meters per second per second" or "meters per second squared." The first way is better at capturing the idea that acceleration is a rate of change of velocity, encouraging the equation,  $v = v_0 + at$ , to make sense intuitively.*

$$x - x_0 = v_0t + \frac{1}{2}at^2$$

*This is the displacement equation. If you are given the initial velocity and the acceleration, for a given time period, you may compute the displacement. Compare this equation with the one just discussed,  $v = v_0 + at$ , and think about calculus for a moment. Velocity is the first derivative of displacement with respect to time. Acceleration, the second derivative, is constant here. Do you see how by differentiating the displacement equation, one can derive the equation for velocity?*

$$v^2 = v_0^2 + 2a(x - x_0)$$

*Each of the four formulas of kinematics allows one to transform knowledge of various given properties of the motion of an object (acceleration, time of travel, initial position, etc.) into knowledge of other properties. A good way to think of any physical formula is as a way of seeing a particular phenomenon so that something new may be learned. This final formula is useful if you do not start in the given information with the duration of motion ( $\Delta t$ ).*

## Vectors and Scalars

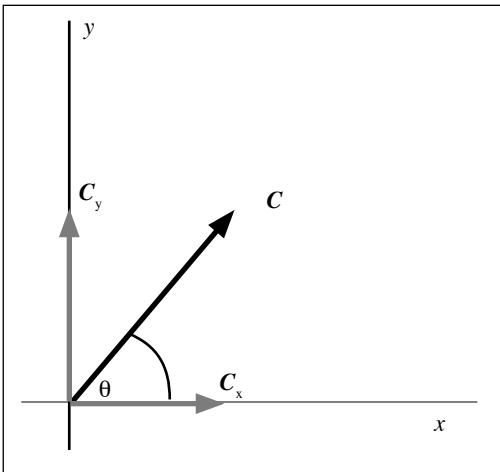
Before we expand our discussion of kinematics to include the case of motion in two dimensions, we must distinguish between vector and scalar quantities. Vectors are physical quantities which must be specified by both magnitude and direction, while scalars have only magnitude and no direction. Examples of vector quantities include displacement, velocity, and force. Examples of scalar quantities include temperature, volume and mass.

A vector quantity is typically represented by an arrow, the length of which describes the magnitude of the quantity and whose orientation corresponds to the quantity's direction. For example, the force of gravity sustaining the orbit of the moon about the earth may be represented in this way:



### Components of a Vector and Vector Addition

In performing operations upon a vector quantity, it is frequently necessary to resolve the vector into components parallel to the axes of a cartesian plane, known as the rectangular components.



The magnitudes of these component vectors,  $C_x$  and  $C_y$ , can be found using simple trigonometry:

$$C_y = C \sin \theta$$

$$C_x = C \cos \theta$$

The magnitude of the original vector can be recovered using the formula:

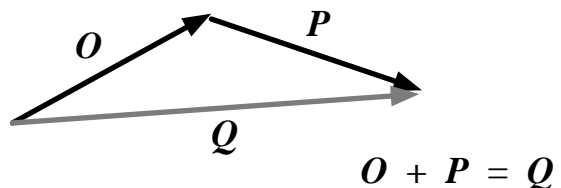
$$C = \sqrt{C_x^2 + C_y^2}$$

*(Note that vectors are generally expressed in bold-face while their magnitudes, along with other scalar quantities, are expressed in plain, italics font.)*

To accomplish the graphical addition of vectors simply place them head to tail and connect the tail of one to the head of the other, giving the resultant vector as illustrated below. To add two vectors numerically, break them down into rectangular components and add the components:

$$O_x + P_x = Q_x$$

$$O_y + P_y = Q_y$$

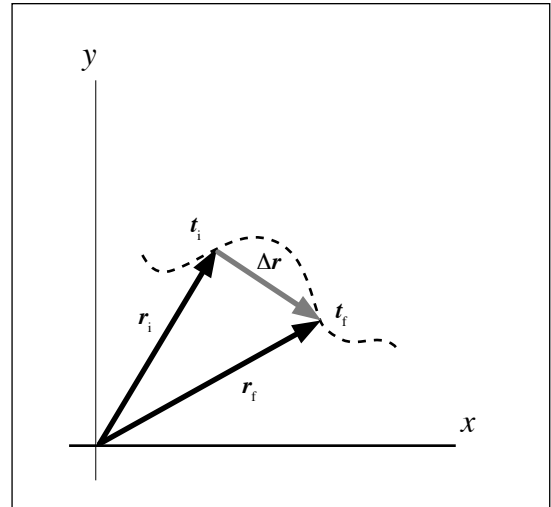


## Motion in Two Dimensions

The displacement vector,  $\mathbf{r}$ , represents the position of a particle moving in the  $xy$  plane at right. The displacement of the particle in the time interval  $\Delta t = t_f - t_i$  is equal to the vector  $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$ .

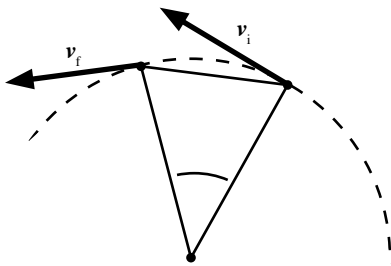
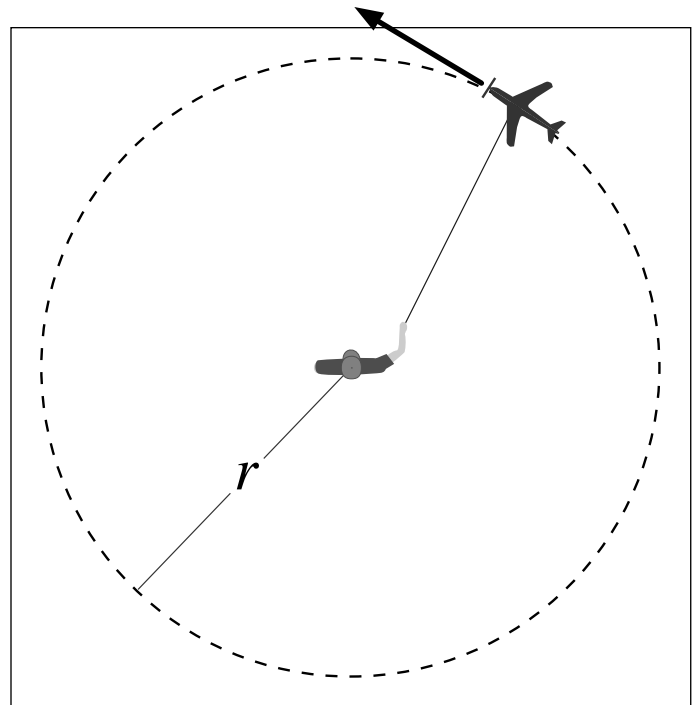
The average velocity equals the amount of change in the displacement vector divided by the time:

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$$



### Uniform Circular Motion

If you imagine the average velocity for an infinitesimally short duration of time, you are conceptualizing the instantaneous velocity. The magnitude of the instantaneous velocity, a vector, is called the speed. Although the toy airplane at right is moving in a circular path at constant speed, it is still undergoing acceleration. Acceleration and velocity are vector quantities. Therefore, a change in the direction of the velocity vector corresponds to an acceleration. Examine the illustration below:



While the magnitude of the velocity remains constant, the direction of the velocity has changed between  $\mathbf{v}_i$  and  $\mathbf{v}_f$ .

An object undergoing uniform circular motion possesses a vector acceleration directed towards the center of the circle with a magnitude given by the formula:

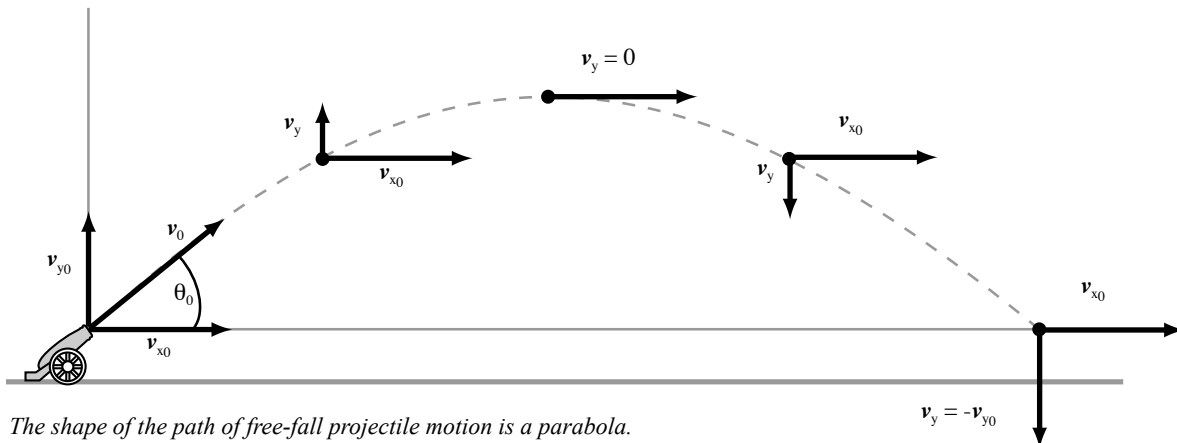
$$a_r = \frac{v^2}{r}$$

An acceleration of this type is referred to as a centripetal acceleration.

## Projectile Motion

Assuming the effect of air resistance to be negligible, a projectile in motion is subject to acceleration only due to gravity, constant over the range of motion and directed downward. The acceleration due to gravity,  $g$ , on or near the surface of the earth is approximately  $9.8 \text{ m/s}^2$ . For most problem solving, an even more approximate figure will do:

$$g = 10 \frac{\text{m}}{\text{s}^2}$$



### Resolving the Motion of a Projectile

A projectile leaves the origin with velocity  $v_0$ , a velocity vector making an angle  $\theta_0$  with the horizontal. Only causing a downward acceleration, gravity affects only the vertical component of the velocity of the projectile  $v_y$ . The horizontal component  $v_x$  is constant for the entire trajectory, therefore, always equal to the initial horizontal velocity  $v_{x0}$ . Utilizing the equations of kinematics one may formulate expressions describing the motion of the projectile in terms of its rectangular components.

$$v_x = v_{x0} = \text{constant} \quad v_y = v_{y0} - gt$$

Assuming air resistance to be negligible, the only acceleration on a projectile in flight is due to gravity:  $g = 10 \frac{\text{m}}{\text{s}^2}$ . The acceleration due to gravity only affects the vertical component of the velocity. The horizontal component is constant for the entire trajectory.

The key to many projectile motion problems is knowing that the vertical velocity is zero at the peak. Knowing the initial vertical component of the velocity and the acceleration due to gravity ( $10 \text{ m/s}^2$  ( $9.8 \text{ m/s}^2$ )), we can determine how long it takes the projectile to reach the peak. The time needed by gravity to deplete the initial vertical velocity to zero is the time to the peak,  $t_p$ .

$$t_p = \frac{v_{y0}}{g}$$

Now, it is a simple matter to find the height of the peak using the time to reach the peak,  $t_p$ , in the formula  $y = v_{y0}t - \frac{1}{2}gt^2$  and the range using the formula  $x = v_{x0}t$  with the time to land exactly double the time to reach the peak.